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New diverse exact optical solutions of the three dimensional Zakharov–Kuznetsov equation

Emad H. M. Zahran[1] · R. A. Ibrahim[1] · Dilber Uzun Ozsahin[2,3,4] · Hijaz Ahmad[4,5,6] · Maha S. M. Shehata[7]

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Abstract
In our current article, we establish four new types of group for the optical soliton solutions to the three dimensional modified Zakharov–Kuznetsov equation's new model. Four distinct and impressive techniques are used for this target, which are the extended simple equation method (ESEM), the extended direct algebraic method (EDAM), the $\left(\frac{G'}{G}\right)$—expansion method and the Paul-Painleve approach method (PPAM). The four suggested techniques are discovered recently to derive the exact solutions for many other NLPDE that arising in various branches of science and usually give good results. These techniques have been used for the first time to extract the soliton solutions of this model. The four employed methods are applied serially and in the same time. Our solutions weren't achieved previously via any other authors who applied any other methods.

Keywords The three dimension MZKE · The ESEM · The EDAM · The $\left(\frac{G'}{G}\right)$-method · The PPAM · The optical solutions

✉ Hijaz Ahmad
hijaz555@gmail.com

[1] Departments of Basic Science, Benha University, Faculty of Engineering, Shubra, Egypt

[2] Department of Medical Diagnostic Imaging, College of Health Science, University of Sharjah, Sharjah, UAE

[3] Research Institute for Medical and Health Sciences, University of Sharjah, Sharjah, UAE

[4] Operational Research Centre in Healthcare, Near East University, TRNC Mersin 10, Nicosia 99138, Turkey

[5] Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon

[6] Section of Mathematics, International Telematic University Uninettuno, Corso Vittorio, Emanuele II, 39, 00186 Roma, Italy

[7] Departments of Mathematics, Zagazig University, Faculty of Science, Zagazig 44519, Egypt

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1 Introduction

The Zakharov–Kuznetsov equation is considered one of important models in plasma physics that definitely describes propagating of nonlinear ion acoustic waves in magnetized plasma that contains hot electrons at constant temperature and cold ions waves in terms of their waves amplitudes. Specially, we will discuss the accurate case in which the electrons are not isothermal that generates various visions to this model namely the ZKE, the MZKE and the generalized ZKE.

In the last few decades many authors studied various forms of the suggested equation see for example Munro and Parkes (1999) who discussed the MZKE via Rowlands Infield method and investigated the stability of the achieved solutions, Sobhanian et al. (2003) who derived various forms of ZK taking in consideration that the electrons are not isothermal, Ahmad and Masood (2015) who discussed the nonlinear propagation of drift acoustic wave, studied the behavior of non-uniform magnetism of plasma mediea and obtained the solution of the modified Zakharov equation via the method of hyperbolic tangent, Naher et al. (2012) who used the developed $\left(\frac{G'}{G}\right)$-expansion method to construct the new traveling wave solutions to the $(2+1)$-dimensional modified Zakharof equation, Eslami et al. (2014) who applied the homogeneous balance technique to extract the traveling wave solutions of MZKE, Yu et al. (2016) who extracted the exact solutions to the initial and generalized forms of ZKE, the MZKE, Zhou (2016) who discussed the nonlinear dynamic to the MZKE in the presence of external force and achieved the soliton behavior the electrical transmission using the Riccati mapping technique, Seadawy (2016a) who studied the $(2+1)$-dimensional MZKE and found the traveling wave solution forms to the electrostatic field potential and electric field and Park et al. (2020a) who studied numerically the dimensionless MZKE using (Cubic & Quantic & Septic) B–spline techniques to establish the accuracy of the calculated computational solutions.

The three dimensions MZKE (Munro and Parkes 1999; Sobhanian et al. 2003; Ahmad and Masood 2015; Naher et al. 2012; Eslami et al. 2014) is:

$$\Upsilon_t + \Upsilon_x \Upsilon^2 + \Upsilon_{xxx} + \Upsilon_{xyy} = 0. \tag{1}$$

Now, if Eq. (1) is submissioned to $\Upsilon(x,t) = R(\zeta)$, $\zeta = x + y - Ct$ it becomes

$$b(\sigma^3 + \sigma\rho^2)R'' + \frac{a\sigma}{2}R^2 - CR = 0. \tag{2}$$

The homogeneous balance for R'', R^2 that appear in Eq. (2) implies $M = 2$.

The main goal of our work is constructing new type of the exact solutions to Eq. (2) the four distinct methods mentioned above that were examined before for other nonlinear problems that arising in applied mathematics and physical science and usually achieve accurate results. The first employed one is the ESEM (Bekir and Zahran 2020) the second one is the EDAM (Seadawy 2016b), the third one is the $\left(\frac{G'}{G}\right)$-expansion method (Bekir et al. 2021) and the fourth one is the PPAM (Kudryashov 2019; Bekir and Zahran 2021). Various studies to the solutions of the NLPDE in different branches of science have been documented in Bekir et al. (2021), Hosseini et al. (2020), Biswas et al. (2010) and Bhrawy (2014), Moreover, there are new techniques to extract the soliton solutions for similar models see for example, Iqbal et al. (2018a) who used extended auxiliary equation mapping and extended direct algebraic methods, to obtain the exact traveling and solitary wave solutions which represent the electrostatic potential to the modified Kortewege–de Vries equation, Lu et al. (2018) who applied the modification form of extended auxiliary equation mapping method to obtain the new exact traveling and solitary wave solutions for the coupled Whitham–Broer–Kaup, (2 + 1)-dimensional Broer–Kaup–Kupershmit and Drinfel'd–Sokolow–Wilson equations, Iqbal et al. (2018b) who used the extend form of two methods, auxiliary equation mapping and direct algebraic methods to obtain the families of dust and ion solitary wave solutions that represent the electrostatic potential and pressure for the one-dimensional nonlinear fmKdV, Seadawy et al. (2019a) who used the extended form of two methods, auxiliary equation mapping and direct algebraic methods to derive the new exact travelling solutions in the form of trigonometric functions, hyperbolic functions, periodic solitary waves, bright and dark solitons, kink solutions for the system of equations for the ion sound and Langmuir waves, Iqbal et al. (2019a) who used the extended form of two methods, auxiliary equation mapping and direct algebraic method to derive the new exact and solitary wave solutions in the form of trigonometric function, periodic solitary wave, rational function, and elliptic function, hyperbolic function, bright and dark solitons solutions for the one-dimensional nonlinear longitudinal wave equation in a magneto-electro-elastic circular rod, Seadawy et al. (2019b) who constructed the bright–dark solitons and traveling wave solutions to the generalized Kadomtsev–Petviashvili equation by using the modification form of extended auxiliary equation mapping method, Seadawy et al. (2020a) who applied the modification form of extended auxiliary equation mapping method to obtain the solitary wave solutions in different form of solitons: the single bright–dark solitons, the double bright–dark solitons and traveling wave solutions to the generalized Kadomtsive–Petviashvili modified equal width-Burgers equation, Iqbal et al. (2019b) who found the solitary wave solutions for the (2 + 1)-dimensional Zakharov–Kuznetsov equation, generalized Zakharov–Kuznetsov equation and generalized form of modified (ZK)-equation by using the modified mathematical method, obtained the bright–dark solitons, traveling wave and solitary wave solutions, Seadawy et al. (2020b) who formulated the exact traveling and solitary wave solutions of nonlinear diffusion reaction equation with quadratic and cubic nonlinearities by using modified mathematical method, obtained new exact traveling and solitary wave solutions in the form of dark solitons, bright solitons, combined dark-bright solitons, traveling wave and periodic wave solutions, Iqbal et al. (2020) who constructed new types of the soliton solutions: rational, trigonometric, hyperbolic, elliptic functions including dark, bright, singular, combined, optical solitons, kink wave, anti-kink wave, periodic wave, travelling wave for the (2 + 1)-dim nonlinear Nizhnik–Novikov–Vesselov equation by using the extended modified rational expansion method, Seadawy et al. (2020c) who formulated nonlinear model for the damped modified Korteweg–de Vries equation arising in ion-acoustic wave in an un-magnetized collisional dusty plasma by applying reductive perturbation technique, constructed a new solitary wave solutions that have the shape of kink wave solitons, antikink wave solitons, half dark–bright solitons, traveling wave, solitary wave and periodic wave by implementing a two techniques on dust ion-acoustic solitary wave nonlinear damped mKdV equation, Seadawy et al. (2019c) who constructed the exact travelling and solitary wave solutions of the Kudryashov–Sinelshchikov equation by implementing the modified mathematical method, obtained new solutions in the shape of hyperbolic, trigonometric, elliptic functions including dark, bright, singular, combined kink wave solitons, travelling wave, solitary wave and periodic wave, Iqbal and Seadawy (2020) who constructed the exact traveling and soliton solutions for unstable nonlinear Schrödinger equation by a modified mathematical method, demonstrated soliton solutions that are represented in bright-dark solitons, periodic solitary wave, and traveling waves and Seadawy et al. (2021) who constructed the new solitary wave solutions for the nonlinear three-dim modified

Kortewege–de Vries Zakharov–Kuznetsov and extended Zakharov–Kuznetsov equationsa by using the modified rational expansion method, obtained new results as solitary waves, periodic waves, traveling waves, kink-antikink solitons, dark-bright solitons.

Furthermore, there are other recent studies for the traveling wave solutions of such equations have been established via some authors see for example, Ali et al. (2021) who used the generalized exponential rational function method to extract abundantly distinct and exact solutions including non-topological solutions, periodic singular solutions, topological solutions, singular solutions, like periodic lump solutions for the strain wave equation in microstructured solids that takes an important place in solid physics is presented for consideration, Ismael et al. (2022a) who used a symbolic computational method to create the Lump solutions and periodic lump waves, the simplified Hirota's method and a long-wave method to create single-, double- and triple-M-lump waves to the $(2+1)$-dimensional Korteweg–de Vries equation with the exten-sion of time-dependent coefficients, Kirci et al. (2021) who used the modified exponential function method to ultimately new hyperbolic, trigonometric and rational forms of the exact solutions for the Lonngren-wave equation and tested whether these forms satisfy this equation or not, Ismael et al. 2022b) who used the $(m+1/G')$-expansion method and the Adomian decomposition method to study the 3D potential Yu–Toda–Sasa–Fukuyama (3D-pYTSF) equation which has a good application in the two-layer liquid medium, established novel exact wave solutions and the numerical solutions for the 3D- pYTSF equation, respectively and Ismael and Bulut (2021) who used the simple Hirota's method study the $(2+1)$-dimensional Sawad–Kotera equation, obtained the features of multi soliton solutions, M-lump waves, the mixed 1-M-lump with one-soliton and two-soliton solutions.

In same connection, there are related articles in this field see for example Akbar et al. (2022) who used the generalized Kudryashov method to extract stable, generic, and wide-ranging soliton solutions, comprising hyperbolic, exponential, trigonometric, and some other functions, and retrieved diverse known soliton structures by balancing the effects of nonlinearity and dispersion to the perturbed nonlinear Schrodinger equation, Osman et al. (2020) who extracted new and further exact soliton solutions in the form dark, bright, combined bright-dark, combined dark-singular, periodic, periodic singular, and elliptic wave solutions to the $(2+1)$-dimensional Heisenberg ferromagnetic spin chain equation via the new extended FAN sub-equation method, Park et al. (2020b) who realized various solitons and solutions of the class of fifth-order Korteweg–de Vries equations by devising suitable novel hyperbolic and exponential ansatze, Nisar et al. (2021) who obtained the analytic solutions for the $(3+1)$-dimensional generalized shallow water-like equation, the $(3+1)$-dimensional Boiti–Leon–Manna–Pempinelli (BLMP) equation, $(3+1)$-dimensional generalized variable-coefficient B-type Kadomtsev–Petviashvili (VC B-type KP) equation and the $(2+1)$-dimensional Caudrey–Dodd–Gibbon–Kotera–Sawada equation by using the multiple Exp-function method. We consider the stated method, Ali et al. (2020) who solved the general form of nonlinear fractional order integro-differential equations (GNFIDEs) with linear functional arguments using Chebyshev series that is presented as a general form to produce delay, proportional delay, and advanced non-linear arbitrary order Fredholm–Volterra integro-differential equations and extended the spectral collocation method to study this problem as a matrix discretization scheme, where the fractional derivatives are characterized in the Caputo sense, Siddique et al. (2021) who obtained the exact traveling solutions of the M-fractional generalized reaction Duffing model and density dependent M-fractional diffusion reaction equation by using three fertile, $(G'/G,\ 1/G)$, modified $\left(\frac{G'}{G^2}\right)$ and $\left(\frac{1}{G'}\right)$ expansion methods, Saliou et al. (2021) who employed the Modified Sardar Sub-Equation Method to extract solitons such as W-shape bright, dark solitons, trigonometric function solutions and singular function solutions to the (3+1)-dimensional nonlinear extended quantum Zakharov–Kuznetsov (NLEQZK) equation and the (3+1)-dimensional nonlinear modified Zakharov–Kuznetsov (NLmZK) equation, respectively, Arafat et al. (2023) who investigated scores of broad-spectral soliton solutions to the two-dimensional nonlinear complex coupled Maccari system via the auxiliary equation technique, obtained solutions that are established as an integration of the rational function, hyperbolic function, trigonometric function and exponential function. Akbar et al. (2023) who executed the (G'/G)-expansion technique to extract diverse soliton solutions for the (1+1)-dimensional Camassa–Holm and Landau–Ginzburg–Higgs equations in suitable forms arrayed to trigonometric, rational, and hyperbolic functions.

2 The ESEM

To investigate this technique, let us introduce firstly the formlism to any NLPDE to be

$$V(R, R_x, R_y, R_t, R_{xx}, R_{yy}, R_{tt}, \ldots) = 0. \tag{3}$$

where V in terms the succssive partial differentiotion to $R(x, y, t)$, when Eq. (3) substentions to the transformation $R(x, y, t) = R(\zeta),\ \zeta = x + y - Ct$, it will be changed to the following ODE

$$\mathrm{Z}(R, R'', R''', \ldots) = 0. \tag{4}$$

where Z is a function in the total derivatives of $R(\zeta)$.

The ESEM proposes the solution of Eq. (4) to be

$$R(\zeta) = \sum_{i=-M}^{M} A_i\ \phi^i(\zeta). \tag{5}$$

where $\phi(\zeta)$ can be determined from

$$\phi'(\zeta) = a_0 + a_1\phi + a_2\phi^2 + +a_3\phi^3. \tag{6}$$

where A_i denotes for constants to be calculated later, while M is integer that will be determined by the balance rule and a_0, a_1, a_2 and a_3 other arbitrary constants for which the following forms of solutions will be generated

(1) When $a_1 = a_3 = 0$, then Eq. (6) represents the Riccati equation that processes the following forms of solutions

$$\phi(\zeta) = \frac{\sqrt{a_0 a_2}}{a_2} \tan(\sqrt{a_0 a_2}(\zeta + \zeta_0), \quad a_0 a_2 > 0. \tag{7}$$

$$\phi(\zeta) = \frac{\sqrt{-a_0 a_2}}{a_2} \tanh\left(\sqrt{-a_0 a_2}\zeta - \frac{\rho_1 \ln \zeta_0}{2}\right), \quad a_0 a_2 < 0, \quad \zeta > 0, \quad \rho_1 = \pm 1. \tag{8}$$

(2) When $a_0 = a_3 = 0$, then Eq. (6) represents the Bernoulli equation that has the following solutions

$$\phi(\zeta) = \frac{a_1\, Exp[a_1(\zeta + \zeta_0)]}{1 - a_2 Exp[a_1(\zeta + \zeta_0)]}\ , \quad a_1 \succ 0. \tag{9}$$

$$\phi(\zeta) = \frac{-a_1\, Exp[a_1(\zeta + \zeta_0)]}{1 + a_2 Exp[a_1(\zeta + \zeta_0)]}\ , \quad a_1 \prec 0. \tag{10}$$

Furthermore, Eq. (4) has the following general forms of solutions

$$\phi(\zeta) = -\frac{1}{a_2}\left(a_1 - \sqrt{4a_1a_2 - a_1^2}\,\tan\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\zeta + \zeta_0)\right)\right), \quad 4a_1a_2 \succ a_1^2, \quad a_2 \succ 0. \tag{11}$$

$$\phi(\zeta) = \frac{1}{a_2}\left(a_1 + \sqrt{4a_1a_2 - a_1^2}\,\tanh\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\zeta + \zeta_0)\right)\right), \quad 4a_1a_2 \succ a_1^2, \quad a_2 \prec 0. \tag{12}$$

where ζ_0 is the constant of integration?

Let us implement this method to Eq. (2) for which the homogeneous implies that $M = 2$, thus the solution is:

$$R(\zeta) = \frac{A_{-2}}{\phi^2} + \frac{A_{-1}}{\phi} + A_0 + A_1\phi + A_2\phi^2. \tag{13}$$

where $\phi' = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3$.

(I) If $a_1 = a_3 = 0 \Rightarrow \phi' = a_0 + a_2\phi^2$, that implies the first family for which the following relations will be detected

$$R' = \frac{-2a_0A_{-2}}{\phi^3} - \frac{a_0A_{-1}}{\phi^2} - \frac{2a_2A_{-2}}{\phi} + (A_1a_0 - a_2A_{-1}) + 2A_2a_0\phi + A_1a_2\phi^2 + 2a_2A_2\phi^3. \tag{14}$$

$$\begin{aligned} R'' = {} & \frac{6a_0^2A_{-2}}{\phi^4} + \frac{2a_0^2A_{-1}}{\phi^3} + \frac{8a_0a_2A_{-2}}{\phi^2} + \frac{2a_0a_2A_{-1}}{\phi} + 2(A_2a_0^2 + A_{-2}a_2^2) \\ & 2a_0a_2A_1\phi + +8a_0a_2A_2\phi^2 + 2a_2^2A_1\phi^3 + 6a_2^2A_2\phi^4. \end{aligned} \tag{15}$$

By inserting the relations (13)–(15) into Eq. (2), collecting the coefficients of different powers of ϕ^i and equating them by zero, we get a system of equation from which the following results will be emerged

(1) $C = -4b\sigma(\sigma^2 + \rho^2)a_0a_2, A_0 = \dfrac{-12b(\sigma^2 + \rho^2)a_0a_2}{a}, A_{-2} = \dfrac{-12b(\sigma^2 + \rho^2)a_0^2}{a}, A_1 = A_{-1} = A_2 = 0.$

(2) $C = 4b\sigma(\sigma^2 + \rho^2)a_0a_2, A_0 = \dfrac{-4b(\sigma^2 + \rho^2)a_0a_2}{a}, A_{-2} = \dfrac{-12b(\sigma^2 + \rho^2)a_0^2}{a}, A_1 = A_{-1} = A_2 = 0.$

$$(3)\quad C=-4b\sigma(\sigma^2+\rho^2)a_0a_2, A_0=\frac{-12b(\sigma^2+\rho^2)a_0a_2}{a}, A_2=\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_1=A_{-1}=A_{-2}=0.$$

$$(4)\quad C=4b\sigma(\sigma^2+\rho^2)a_0a_2, A_0=\frac{-24b(\sigma^2+\rho^2)a_0a_2}{a}, A_2=\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_1=A_{-1}=0.$$

$$(5)\quad C=-16b\sigma(\sigma^2+\rho^2)a_0a_2, A_0=\frac{-12b(\sigma^2+\rho^2)a_0a_2}{a}, A_2 =\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_{-2}=\frac{-12b(\sigma^2+\rho^2)a_0^2}{a}, A_1=A_{-1}=0.$$

$$(6)\quad C=16b\sigma(\sigma^2+\rho^2)a_0a_2, A_0=\frac{8b(\sigma^2+\rho^2)a_0a_2}{a}, A_2=\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_{-2}=\frac{-12b(\sigma^2+\rho^2)a_0^2}{a}, A_1=A_{-1}=0. \tag{16}$$

Now, let us demonstrate the solutions that correspond to only two results of them say the first and the six results.

(1) The first result that can be simplified to become

$$\begin{aligned} &C=-4,\quad A_0=A_{-2}=1,\quad a_0^2=a_2^2=0.5,\\ &a=-12,\quad C=-4,\quad A_1=A_{-1}=A_2=0. \end{aligned} \tag{17}$$

From which either $a_0=a_2=0.7$ or $a_0=a_2=-0.7$ at the same time, thus the solution according to these values is powers of $R(\zeta)=\frac{A_{-2}}{\phi^2}+\frac{A_{-1}}{\phi}+A_0+A_1\phi+A_2\phi^2$, where

$$\phi(\zeta)=\frac{\sqrt{a_0a_2}}{a_2}\tan(\sqrt{a_0a_2}(\zeta+\zeta_0),\ a_0a_2>0.$$

$$\phi(\zeta)=\tan[0.7(\zeta+\zeta_0)]. \tag{18}$$

$$R(\zeta)=1+\cot^2[0.7(\zeta+\zeta_0)]. \tag{19}$$

(B) The sixth result that can be simplified to become

$$C=-24,\quad A_{-2}=A_0=A_2=1,\quad A_1=A_{-1}=0,\quad a_0=\pm 0.7,\quad a_2=\pm 1.1. \tag{20}$$

From which either $a_0=0.7, a_2=-1.1$ or $a_0=-0.7, a_2=1.1$ at the same time, thus the solution according to these values is $R(\zeta)=\frac{A_{-2}}{\phi^2}+\frac{A_{-1}}{\phi}+A_0+A_1\phi+A_2\phi^2$, where

$$\phi(\zeta)=\frac{\sqrt{-a_0a_2}}{a_2}\tanh\left(\sqrt{-a_0a_2}\zeta-\frac{\rho_1\ln\zeta_0}{2}\right),\quad a_0a_2<0,\quad \zeta>0,\quad \rho_1=\pm1.$$

$$\phi(\zeta)=-0.8\tanh(0.9\zeta+0.3). \tag{21}$$

$$R(\zeta)=1.6(\coth(0.9\zeta+0.3))^2+1+0.64(\tanh(0.9\zeta+0.3))^2. \tag{22}$$

(II) If $a_0=a_3=0\Rightarrow\phi'=a_1\phi+a_2\phi^2$, that implies the second family for which the following relations will be detected

$$R'=-\frac{2A_{-2}a_1}{\phi^2}-\frac{2A_{-2}a_2+A_{-1}a_1}{\phi}-A_{-1}a_2+a_1A_1\phi+(A_1a_2+2a_1A_2)\phi^2+2A_2a_2\phi^3. \tag{23}$$

$$\begin{aligned}R''=&\frac{4A_{-2}a_1^2}{\phi^2}+\frac{6A_{-2}a_1a_2+A_{-1}a_1^2}{\phi}+A_1a_1^2\phi+2A_{-2}a_2^2+A_{-1}a_1a_2\\&+(4A_2a_1^2+3a_1a_2A_1)\phi^2+(10a_1a_2A_2+2a_2^2A_1)\phi^3+6a_2^2A_2\phi^4.\end{aligned} \tag{24}$$

By inserting the relations (13) and (23–24) into Eq. (2), implementing the equivalence for various powers of ϕ^i and by solving the emerged system we get

$$\begin{aligned}&(1)\quad C=b\sigma(\sigma^2+\rho^2)a_1^2, A_1=\frac{-12b(\sigma^2+\rho^2)a_1a_2}{a}, A_2=\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_0=A_{-1}=A_{-2}=0.\\&(2)\quad C=-b\sigma(\sigma^2+\rho^2)a_1^2, A_0=\frac{-2(\sigma^2+\rho^2)a_1^2}{a}, A_1=\frac{-12b(\sigma^2+\rho^2)a_1a_2}{a}, A_2=\frac{-12b(\sigma^2+\rho^2)a_2^2}{a}, A_{-1}=A_{-2}=0.\end{aligned} \tag{25}$$

Let us only implement the corresponding solution for the first result that can be simplified to be

$$A_0=A_{-1}=A_{-2}=0,\quad C=A_1=A_2=1,\quad a_1=a_2=\pm0.7,\quad a_1a_2=0.5. \tag{26}$$

The solution is $R(\zeta)=\frac{A_{-2}}{\phi^2}+\frac{A_{-1}}{\phi}+A_0+A_1\phi+A_2\phi^2$, where ϕ can be determined from

$$\begin{aligned}\phi(\zeta)&=\frac{a_1\,Exp[a_1(\zeta+\zeta_0)]}{1-a_2Exp[a_1(\zeta+\zeta_0)]},\quad a_1>0.\\\phi(\zeta)&=\frac{0.7\,Exp[0.7(\zeta+\zeta_0)]}{1-0.7Exp[0.7(\zeta+\zeta_0)]}.\end{aligned} \tag{27}$$

$$R(\zeta)=\frac{0.7\,Exp[0.7(\zeta+\zeta_0)]}{1-0.7Exp[0.7(\zeta+\zeta_0)]}+\left(\frac{0.7\,Exp[0.7(\zeta+\zeta_0)]}{1-0.7Exp[0.7(\zeta+\zeta_0)]}\right)^2. \tag{28}$$

By the same vision we can design the soltion solution of the other result.

3 The EDAM

The solution of Eq. (4) according to this method is:

$$R(\zeta)=\sum_{i=0}^{M} b_i\ \phi^i(\zeta),\quad \phi'^2=\alpha\phi^2+\beta\phi^3+\gamma\phi^4 \tag{29}$$

When we put $M=2$ in the above equation we get

$$R(\zeta)=b_0+b_1\phi+b_2\phi^2. \tag{30}$$

Hence

$$R'=b_1\phi'+2b_2\phi\phi'. \tag{31}$$

$$R''=b_1\phi''+2b_2\phi'^2+2b_2\phi\phi''. \tag{32}$$

$$\phi'^2=\alpha\phi^2+\beta\phi^3+\gamma\phi^4. \tag{33}$$

$$\phi''=\alpha\phi+1.5\beta\phi^2+2\gamma\phi^3. \tag{34}$$

Via introducing the relations (30)–(34) into Eq. (2), collecting the coefficients of different powers of ϕ^i and equating them to zero, this lead to a system of equations that have the following results

$$\begin{aligned}
&(1)\quad \alpha=\frac{C(0.3-0.5\sigma^2)}{b\sigma(\sigma^2+\rho^2)},\beta=0,\gamma=\frac{-0.1ab_2}{b(\sigma^2+\rho^2)},b_0=\frac{2C}{a\sigma},b_1=0.\\
&(2)\quad \alpha=\frac{C(1-2\sigma^2)}{b\sigma(\sigma^2+\rho^2)},\beta=\frac{0.6\sqrt{aCb_2(1-2\sigma^2)}}{b\sqrt{\sigma}(\sigma^2+\rho^2)},\gamma=\frac{-0.1ab_2}{b(\sigma^2+\rho^2)},b_0=\frac{2c}{a\sigma},b_1=\frac{-3.5\sqrt{Cb_2(1-2\sigma^2)}}{\sqrt{a\sigma}}.\\
&(3)\quad \alpha=\frac{C(1-2\sigma^2)}{b\sigma(\sigma^2+\rho^2)},\beta=\frac{-0.6\sqrt{aCb_2(1-2\sigma^2)}}{b\sqrt{\sigma}(\sigma^2+\rho^2)},\gamma=\frac{-0.1ab_2}{b(\sigma^2+\rho^2)},b_0=\frac{2c}{a\sigma},b_1=\frac{3.5\sqrt{Cb_2(1-2\sigma^2)}}{\sqrt{a\sigma}}.
\end{aligned} \tag{35}$$

Now, we will derive the solutions of the first two results.

(1) The first result that can be simplified to become

$$a=b=C=\sigma=\rho=1,\quad b_0=2,b_1=0,\quad b_2=2,\quad \alpha=-0.1,\quad \beta=0,\quad \gamma=-0.1. \tag{36}$$

The solution according to the suggested method is $R(\zeta)=2+2\phi^2$, ϕ can be obtained from the relation $\phi'^2=\alpha\phi^2+\gamma\phi^4$ to be

$$R(\zeta)=\frac{2e^{0.3i\zeta}}{1-e^{0.6i\zeta}}=\frac{(\cos 0.3\zeta-\cos 0.9\zeta)+i(\sin 0.3\zeta-\sin 0.9\zeta)}{1-\cos 0.6\zeta}. \tag{37}$$

Hence the solution is:

$$R(\zeta)=2+2\left(\frac{2e^{0.3i\zeta}}{1-e^{0.6i\zeta}}\right)^2.$$

$$R(\zeta)=2+2\left(\frac{(\cos 0.3\zeta-\cos 0.9\zeta)+i(\sin 0.3\zeta-\sin 0.9\zeta)}{1-\cos 0.6\zeta}\right)^2. \tag{38}$$

$$\mathrm{Re}\,R(\zeta) = 2 + \left(\frac{2\cos 0.6\zeta + 2\cos 1.8\zeta - 4\cos 1.2\zeta)}{1 - 2\cos 0.6\zeta + \cos^2 0.6\zeta}\right). \tag{39}$$

$$\mathrm{Im}R(\zeta) = 2\left(\frac{\cos 0.3\zeta \sin 0.3\zeta - \sin 1.2\zeta + \sin 0.9\zeta \cos 0.9\zeta}{1 - 2\cos 0.6\zeta + \cos^2 0.6\zeta}\right). \tag{40}$$

(B) The second result that can be simplified to become

$$a = b = C = \sigma = \rho = 1, \quad b_0 = 2, \quad b_1 = -2.5, \quad b_2 = -2, \quad \alpha = -0.5, \quad \beta = 0.2, \quad \gamma = 0.1. \tag{41}$$

Consequently, the solution is $R(\zeta) = 2 + 2\phi^2$, ϕ can be obtained from the relation $\phi'^2 = \alpha\phi^2 + \gamma\phi^4$ to be

$$\phi = \frac{1}{\sin 0.3\zeta - 0.1}. \tag{42}$$

Hence, the solution is:

$$R(\zeta) = 2 - 2.5\left(\frac{1}{\sin 0.3\zeta - 0.1}\right) - 2\left(\frac{1}{\sin 0.3\zeta - 0.1}\right)^2. \tag{43}$$

4 The mathematical analysis of the (G′/G)-expansion method

The solution of Eq. (4) according to this method is:

$$R(\zeta) = A_0 + \sum_{k=1}^{M} A_k \left[\frac{G'}{G}\right]^k, \quad A_M \neq 0. \tag{44}$$

where M is the balance number, the function $G(\zeta)$ achieves the auxiliary equation $G'' + \mu G' + \lambda G = 0$ whose solutions forms depend on the discriminate $\mu^2 - 4\lambda$.

(I) If $\mu^2 - 4\lambda > 0$ the solution is:

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\mu^2 - 4\lambda}}{2}\left(\frac{l_1 \sinh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cosh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta}{l_1 \cosh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \sinh\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta}\right) - \frac{\mu}{2}. \tag{45}$$

(II) If $\mu^2 - 4\lambda < 0$ the solution is:

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\mu^2 - 4\lambda}}{2}\left(\frac{-l_1 \sin\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cos\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta}{l_1 \cos\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \sin\left(\frac{\sqrt{\mu^2-4\lambda}}{2}\right)\zeta}\right) - \frac{\mu}{2}. \tag{46}$$

(III) If $\mu^2 - 4\lambda = 0$ the solution is:

$$\left(\frac{G'}{G}\right) = \left(\frac{l_2}{l_1 + l_2\zeta}\right) - \frac{\mu}{2}. \tag{47}$$

where $l_1,\ l_2$ are constants.

Now, for the proposed model Eq. (2) whose balance number is $M = 2$ the solution is:

$$R(\zeta) = A_0 + A_1\left(\frac{G'}{G}\right) + A_2\left(\frac{G'}{G}\right)^2. \tag{48}$$

$$\begin{aligned} R'' = 6A_2\left(\frac{G'}{G}\right)^4 + (2A_1 + 10\mu A_2)\left(\frac{G'}{G}\right)^3 + (3A_1\mu + 8A_2\lambda + 2\mu A_2)\left(\frac{G'}{G}\right)^2 \\ +(2A_1\lambda + A_1\mu^2 + 6A_2\lambda\mu)\left(\frac{G'}{G}\right) + A_1\lambda\mu + 2A_2\lambda^2. \end{aligned} \tag{49}$$

By inserting the above two relation into the proposed Eq. (2), collecting the coefficients of different powers of $\left(\frac{G'}{G}\right)^i$ and equating them to zero, this lead to a system of equations that have the following results

$$\begin{aligned}
&(1)\quad A_0 = \frac{3C}{a\sigma}, A_1 = 0, A_2 = \frac{-12b(\sigma^2+\rho^2)}{a}, \mu = 0, \lambda = \frac{-C}{4b\sigma(\sigma^2+\rho^2)}.\\
&(2)\quad A_0 = \frac{-C}{a\sigma}, A_1 = 0, A_2 = \frac{-12b(\sigma^2+\rho^2)}{a}, \mu = 0, \lambda = \frac{C}{4b\sigma(\sigma^2+\rho^2)}.\\
&(3)\quad A_0 = \frac{-3[-4C+(\sigma^2+\rho^2)]}{4a\sigma}, A_1 = \frac{-6b(\sigma^2+\rho^2)}{a}, A_2 = \frac{-12b(\sigma^2+\rho^2)}{a}, \mu = \frac{1}{2}, \lambda = \frac{-4C+b\sigma(\sigma^2+\rho^2)}{16b\sigma(\sigma^2+\rho^2)}.\\
&(4)\quad A_0 = \frac{-4C+3b\sigma(\sigma^2+\rho^2)]}{4a\sigma}, A_1 = \frac{-6b(\sigma^2+\rho^2)}{a}, A_2 = \frac{-12b(\sigma^2+\rho^2)}{a}, \mu = \frac{1}{2}, \lambda = \frac{4C+b\sigma(\sigma^2+\rho^2)}{16b\sigma(\sigma^2+\rho^2)}.
\end{aligned} \tag{50}$$

For the symmetry and similarity of these results we will extract the solutions corresponding to the second and third result only.

(1) The first result that can be simplified to become

$$a = b = C = \sigma = \rho = 1, A_0 = -1, A_1 = 0, A_2 = -24, \mu = 0, \lambda = 0.13. \tag{51}$$

The solution according to this result is $R(\zeta) = A_0 + A_1\left(\frac{G'}{G}\right) + A_2\left(\frac{G'}{G}\right)^2$, where

$$\begin{aligned} \left(\frac{G'}{G}\right) &= 0.4i\left(\frac{-\sin 0.4i\zeta + 2\cos 0.4i\zeta}{\cos 0.4i\zeta + 2\sin 0.4i\zeta}\right). \\ \left(\frac{G'}{G}\right) &= \left(\frac{2.3\sinh 0.4\zeta\cosh 0.4\zeta + i\,(0.8\cosh^2 0.4\zeta - 1.4\sinh^2 0.4\zeta)}{\cosh^2 0.4\zeta + 4\sinh^2 0.4\zeta}\right). \end{aligned} \tag{52}$$

$$R(\zeta) = -1 - 24\left(\frac{2.3\sinh 0.4\zeta\cosh 0.4\zeta + i\,(0.8\cosh^2 0.4\zeta - 1.4\sinh^2 0.4\zeta)}{\cosh^2 0.4\zeta + 4\sinh^2 0.4\zeta}\right)^2 \tag{53}$$

$$\operatorname{Re} R(\zeta) = -1 + \left(\frac{15.4\cosh^4 0.4\zeta + 47\sinh^4 0.4\zeta - 180\sinh^2 0.4\zeta\cosh^2 0.4\zeta}{\cosh^4 0.4\zeta + 16\sinh^4 0.4\zeta + 8\sinh^2 0.4\zeta\cosh^2 0.4\zeta} \right). \tag{54}$$

$$\operatorname{Im} R(\zeta) = -\left(\frac{110.4\sinh 0.4\zeta\cosh 0.4\zeta\,(0.8\cosh^2 0.4\zeta - 1.4\sinh^2 0.4\zeta)}{\left(\cosh^2 0.4\zeta + 4\sinh^2 0.4\zeta\right)^2} \right). \tag{55}$$

(B) The second result that can be simplified to become

$$a = b = C = \sigma = \rho = 1, \quad A_0 = 0.5, \quad A_1 = -12, \quad A_2 = -24, \quad \mu = 0.5, \quad \lambda = 0.2. \tag{56}$$

The solution for this result is $R(\zeta) = A_0 + A_1\left(\frac{G'}{G}\right) + A_2\left(\frac{G'}{G}\right)^2$, where

$$\left(\frac{G'}{G}\right) = \frac{0.4\sinh 0.4\zeta + 0.8\cosh 0.4\zeta}{\cosh 0.4\zeta + 2\sinh 0.4\zeta} - 0.25.$$

$$\begin{aligned} R(\zeta) = 1.5 &- \left(\frac{4.8\sinh 0.4\zeta + 9.6\cosh 0.4\zeta}{\cosh 0.4\zeta + 2\sinh 0.4\zeta} - 3 \right) \\ &- 24\left(\frac{0.4\sinh 0.4\zeta + 0.8\cosh 0.4\zeta}{\cosh 0.4\zeta + 2\sinh 0.4\zeta} - 0.25 \right)^2 \end{aligned} \tag{57}$$

5 The PPAM

The solution of Eq. (4) according to this method is:

$$\begin{cases} R(\zeta) = A_0 + A_1 W(X)\, e^{-M\zeta}, \quad X = S(\zeta)\,. \\ R(\zeta) = A_0 + A_1 W(X)\, e^{-M\zeta} + A_2 W^2(X)\, e^{-2M\zeta}, \quad X = S(\zeta)\,. \end{cases} \tag{58}$$

where $W(X)$ can be derived from the auxiliary equation "Riccati-equation" $W_X - AW^2 = 0$, to be

$$W(X) = \frac{1}{AX + X_0}, \quad X = S(\zeta) = D - \frac{e^{-N\zeta}}{N}. \tag{59}$$

Now, for the proposed model Eq. (2) whose balance number is $M = 2$ the solution is:

$$R(\zeta) = A_0 + A_1 W e^{-M\zeta} + A_2 W^2\, e^{-2M\zeta}.$$

Consequently,

$$R' = -Ne^{\zeta} W - (A + 2N)e^{-2N\zeta} W^2 - 2Ae^{-3N\zeta} W^3 \tag{60}$$

$$R'' = -6A^2 e^{-4N\zeta} W^4 + (2A^2 + 10AN)e^{-3N\zeta} W^3 + (3AN + 2N^2)e^{-2N\zeta} W^2 + N^2 e^{-N\zeta} W \tag{61}$$

Now, inserting the relation (60)–(61) into the proposed Eq. (2), collecting the coefficients of different powers of $W^i e^{-iN\zeta}$ and equating them to zero, this lead to a system of equations that have the following results

$$\begin{aligned}
&(1)\quad \rho = -i\sigma\sqrt{\frac{17}{23}}, A = \frac{-i\sqrt{\frac{23a}{6}}}{2\sigma\sqrt{b}}, N = \frac{-i\sqrt{\frac{23a}{6}}}{10\sigma\sqrt{b}}.\\
&(2)\quad \rho = i\sigma\sqrt{\frac{17}{23}}, A = \frac{-i\sqrt{\frac{23a}{6}}}{2\sigma\sqrt{b}}, N = \frac{-i\sqrt{\frac{23a}{6}}}{10\sigma\sqrt{b}}.\\
&(3)\quad \rho = -i\sigma\sqrt{\frac{17}{23}}, A = \frac{i\sqrt{\frac{23a}{6}}}{2\sigma\sqrt{b}}, N = \frac{i\sqrt{\frac{23a}{6}}}{10\sigma\sqrt{b}}.\\
&(4)\quad \rho = i\sigma\sqrt{\frac{17}{23}}, A = \frac{i\sqrt{\frac{23a}{6}}}{2\sigma\sqrt{b}}, N = \frac{i\sqrt{\frac{23a}{6}}}{10\sigma\sqrt{b}}.
\end{aligned} \tag{62}$$

For symmetry and similarity of these results we will extract the solution corresponding to the first result that can be simplified to be

$$\rho = -0.9i, \quad a = b = D = X_0 = \sigma = 1, \quad A = -i, \quad N = 0.2i. \tag{63}$$

Thus the solution is

$$R(\zeta) = \frac{e^{0.2i\zeta}}{i(1 + \frac{e^{0.2\zeta}}{0.2i}) + 1}. \tag{64}$$

$$\operatorname{Re} R(\zeta) = \frac{0.04\cos 0.2\zeta + 0.2\cos 0.4\zeta - 0.04\sin 0.2\zeta}{1.08 + 0.4\cos 0.2\zeta + 0.4\sin 0.2\zeta}. \tag{65}$$

$$\operatorname{Im} R(\zeta) = \frac{-0.04\cos 0.2\zeta - 0.04\sin 0.2\zeta - 0.2\sin 0.4\zeta}{1.08 + 0.4\cos 0.2\zeta + 0.4\sin 0.2\zeta} \tag{66}$$

Similarly, we can derive the soliton behaviors of the other results.

6 Conclusion

Through our current article, four new types of group's for the optical soliton solution to the three dimension Zakharov–Kuznetsov equation have been documented via four distinct and impressive techniques. The first group of solutions that are demonstrated by using the ESEM which are periodic trigonometric solutions and hyperbolic solutions Figs. 1, 2, and 3, the second group of solutions that are established by using the EDAM which are the periodic M-shaped solutions, periodic W-like shaped solutions and periodic parabolic solutions Figs. 4, 5 and 6, the third group of solutions that are demonstrated by using the (G'/G)-method which are the new rational solutions Figs. 7, 8 and 9 and the fourth group of solutions that established by using the PPAM which are the periodic trigonometric

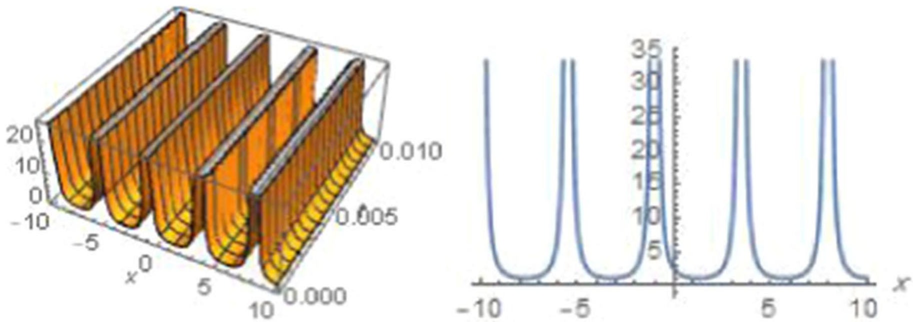

Fig. 1 The 2D and 3D designs of the lump solution Eq. (19) with values: $C=-4, A_0=A_{-2}=1, a_0^2=a_2^2=0.5, a=-12, C=-4, A_1=A_{-1}=A_2=0$

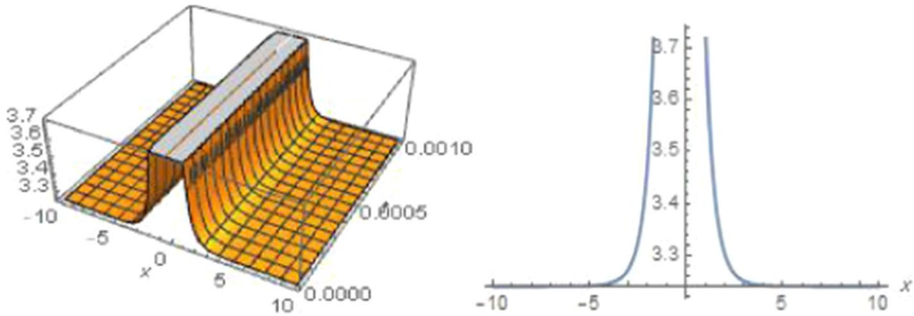

Fig. 2 The 2D and 3D designs of the lump solution Eq. (19) Eq. (22) with values: $C=-24, a=b=A_{-2}=A_0=A_2=1, A_1=A_{-1}=0, a_0=0.7, a_2=-1.1, \rho_1=-1, \zeta_0=2$

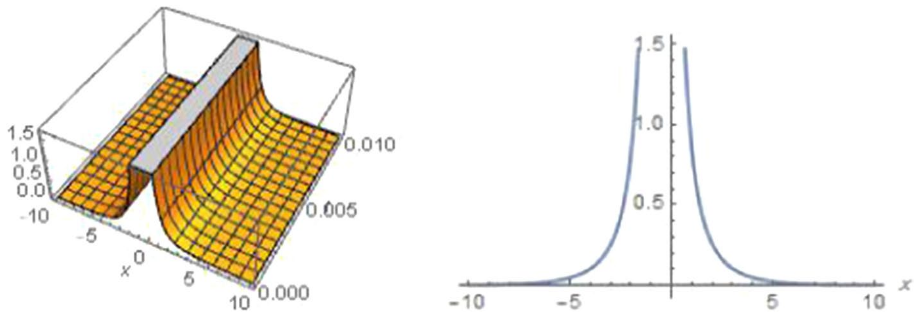

Fig. 3 The 2D and 3D designs of the lump solution Eq. (28) with values: $A_0=A_{-1}=A_{-2}=0, a=b=C=A_1=A_2=1, a_1=0.7, a_2=0.7, a_1a_2=0.5$

solutions Figs. 10 and 11. Many new behaviors of the soliton solutions as M-shape, W-like shape, periodic trigonometric function shape, parabolic function shape and other rational form solutions have been predicted. Most of the achieved optical solutions through these four schemas are new with respect to that achieved before by Yu et al. (2016), Zhou (2016), Seadawy (2016a), Park et al. (2020a). Furthermore, the obtained results will add new coming visions for all dependent phenomena. The achieved results showed that the used

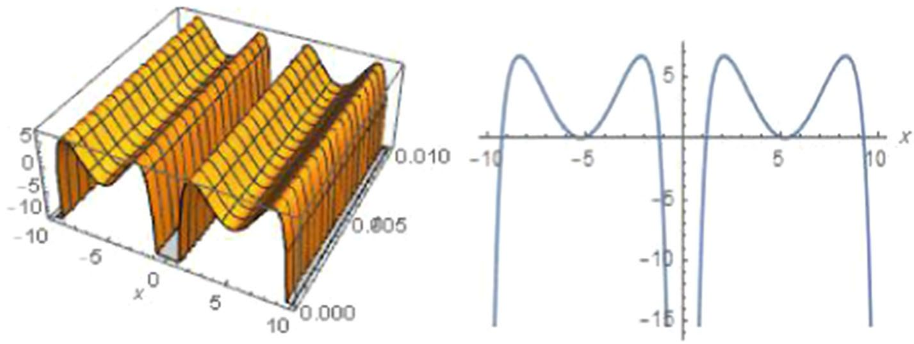

Fig. 4 The 2D and 3D soliton behaviors to the Re Part Eq. (39) with values: $a = b = C = \sigma = \rho = 1, b_0 = 2, b_1 = 0, b_2 = 2, \alpha = -0.1, \beta = 0, \gamma = -0.1$

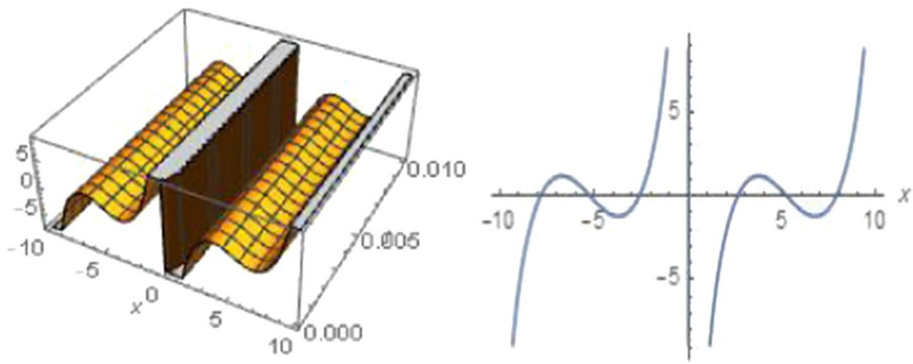

Fig. 5 The 2D and 3D soliton behaviors to the Im. Part Eq. (40) with values: $a = b = C = \sigma = \rho = 1, b_0 = 2, b_1 = 0, b_2 = 2, \alpha = -0.1, \beta = 0, \gamma = -0.1$

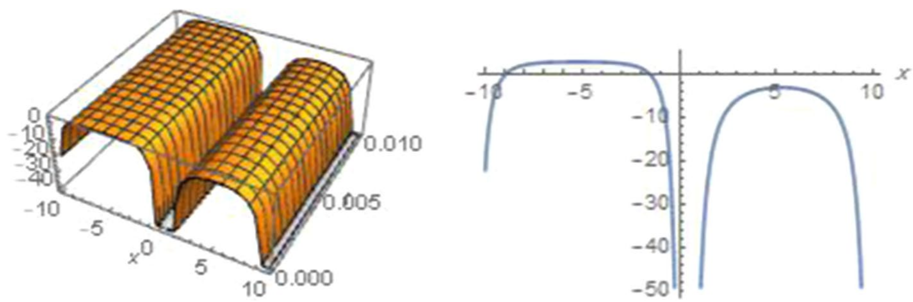

Fig. 6 The 2D and 3D soliton behaviors to Eq. (43) with values: $a = b = C = \sigma = \rho = 1, b_0 = 2, b_1 = -2.5, b_2 = -2, \alpha = -0.5, \beta = 0.2, \gamma = 0.1$

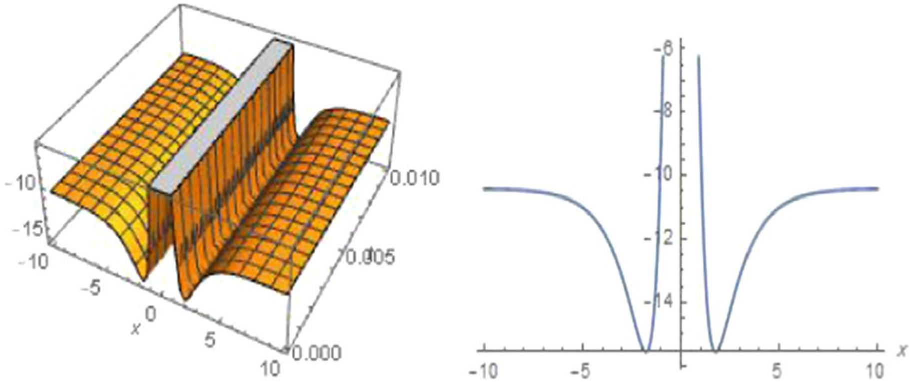

Fig. 7 The 2D and 3D soliton behaviors to the Re Part Eq. (54) when: $a = b = C = \sigma = \rho = 1, A_0 = -1, A_1 = 0, A_2 = -24, \mu = 0, \lambda = 0.13, l_1 = 1, l_2 = 2$

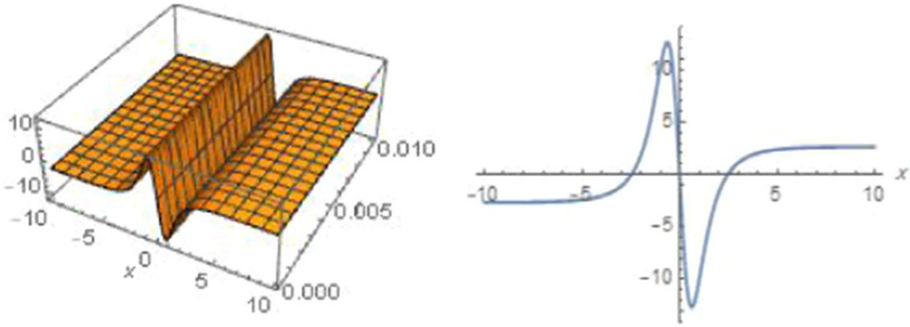

Fig. 8 The 2D and 3D soliton behaviors to the Im. Part Eq. (55) when: $a = b = C = \sigma = \rho = 1, A_0 = -1, A_1 = 0, A_2 = -24, \mu = 0, \lambda = 0.13, l_1 = 1, l_2 = 2$

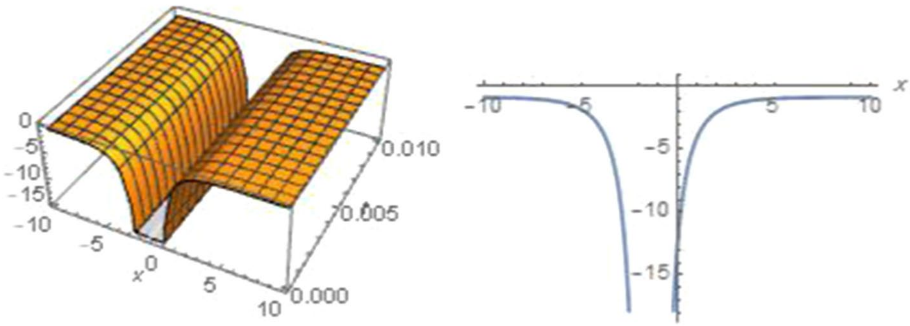

Fig. 9 The 2D and 3D soliton behaviors to Eq. (57) when: $a = b = C = \sigma = \rho = 1, A_0 = 0.5, A_1 = -12, A_2 = -24, \mu = 0.5, \lambda = 0.2, l_1 = 1, l_2 = 2$

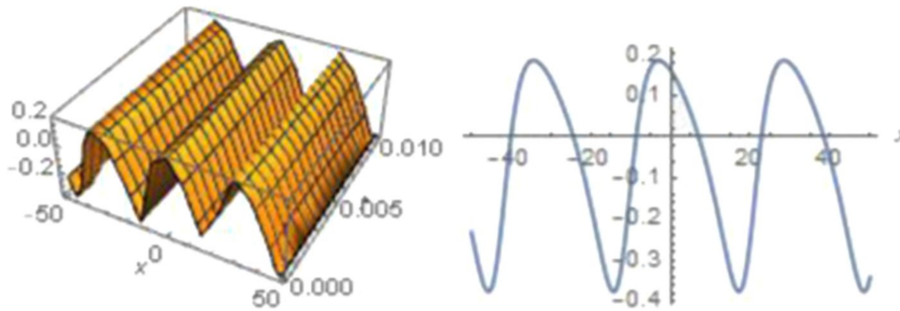

Fig. 10 The 2D and 3D soliton behaviors to the Re. Part Eq. (65) when: $\rho = -0.9i\,, C = a = b = D = X_0 = \sigma = 1\,, A = -i, N = 0.2i$

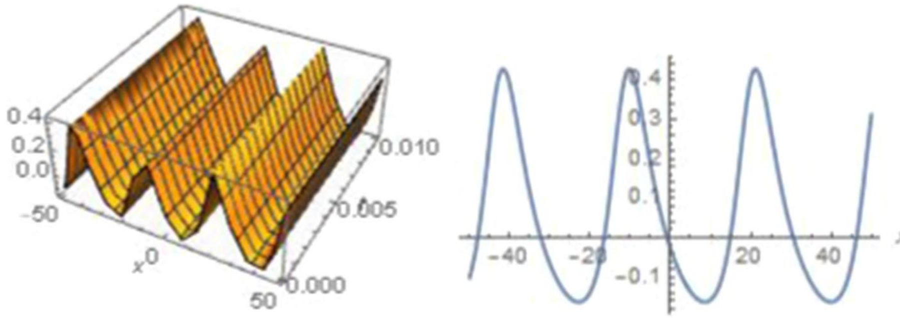

Fig. 11 The 2D and 3D soliton behaviors to the Im. Part Eq. (66) when: $\rho = -0.9i\,, C = a = b = D = X_0 = \sigma = 1\,, A = -i, N = 0.2i$

rational schemes are straightforward, effectual and is a more potent tool. By contrasting our findings with those of other studies that have been published and address such problems or subjects, the realizing of our research was demonstrated.

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Declarations

Competing interests The authors declare no competing interests.

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Ahmad, A., & Masood, W. (2015). Modified Zakharov–Kuznetsov equation for a non-uniform electron–positron–ion magnetoplasma with kappa-distributed electrons. J. Plasma Phys, **81**(5), 905810505. https://doi.org/10.1017/S0022377815000641

Akbar, M.A., Wazwaz, A.M., Mahmud, F., Baleanu, D., Roy, R., Barman, H.K., Mahmoud, W., AlSharif, M.A., Osman, M.S.: Dynamical behavior of solitons of the perturbed nonlinear Schrodinger equation and microtubules through the generalized Kudryashov scheme. Result Phys. **43**, 106079 (2022)

Akbar, M.A., Abdullah, F.A., Islam, M.T., Al Sharif, M.A., Osman, M.S.: New solutions of the soliton type of shallow water waves and superconductivity models. Results Phys. **44**, 106180 (2023)

Ali, K.K., Abd Elsalam, M.A., Mohamed, E.M.H., Samet, B., Kumar, S., Osman, M.S.: Numerical solution for generalized nonlinear fractional integro-differential equations withlinear functional arguments using Chebyshev series. Adv. Differ. Equ. **1**, 494 (2020)

Ali, K.K., Yilmazr, R., Bulut, H., Akturk, T., Osman, M.S.: Abundant exact solutions to the strain wave equation in micro-structured solids. Modern Phys. Lett. B **35**(26), 2150439 (2021)

Arafat, S.M.Y., Fatema, K., Islam, S.M.R., Islam, M.E., Akbar, M.A., Osman, M.S.: The mathematical and wave profile analysis of the Maccari system in nonlinear physical phenomena. Opt. Quant. Electron. **55**(2), 136 (2023)

Bekir, A., Zahran, E.H.M.: Bright and dark soliton solutions for the complex Kundu-Eckhaus equation. Opt. Int. J. Light Electron Opt. **223**, 165233 (2020)

Bekir, A., Zahran, E.H.M.: Optical soliton solutions of the thin-film ferro-electric materials equation according to the Painlevé approach. Opt. Quantum Electron. **53**, 118 (2021)

Bekir, A., Zahran, E.M.H., Shehata, M.S.M.: Comparison between the new exact and numerical solutions of the Mikhailov–Novikov–Wang equation. Numer. Method. Part. Differ. Equ. J. (2021). https://doi.org/10.1002/num.22775

Bhrawy, A.H.: An efficient Jacobi pseudo spectral approximation for nonlinear complex generalized Zakharov system. Appl. Math. Comput **247**, 30–46 (2014)

Biswas, A., Zerrad, E., Gwanmesia, J., Khouri, R.: 1-soliton solution of the generalized Zakharov equation in plasmas by he's variational principle. Appl. Math. Comput. **215**, 4462–4466 (2010)

Eslami, M., Vajargah, B.F., Mirzazadeh, M.: Exact solutions of modified Zakharov–Kuznetsov equation by the homogeneous balance method. Ain Shams Eng. J. **5**, 221–225 (2014)

Hosseini, K., Osman, M.S., Mirzazadeh, M., Rabiei, F.: Investigation of different wave structures to the generalized third-order nonlinear Schrödinger equation. Optik **206**, 164259 (2020)

Iqbal, M., Seadawy, A.: Instability of modulation wave train and disturbance of time period in slightly stable media for unstable nonlinear Schrödinger dynamical equation. Mod. Phys. Lett. B **34**(supp01), 2150010 (2020)

Iqbal, M., Seadawy, A., Lu, D.: Construction of solitary wave solutions to the nonlinear modified Kortewege–de Vries dynamical equation in un-magnetized plasma via mathematical methods. Modern Phys. Lett. A **33**(32), 1850183 (2018a)

Iqbal, M., Seadawy, A., Lu, D.: Dispersive solitary wave solutions of nonlinear further modified Korteweg–de Vries dynamical equation in an un-magnetized dusty plasma. Modern Phys. Lett. A **33**(37), 1850217 (2018b)

Iqbal, M., Seadawy, A., Lu, D.: Applications of nonlinear longitudinal wave equation in a magneto-electro-elastic circular rod and new solitary wave solutions. Mod. Phys. Lett. B **33**(18), 1950210 (2019a)

Iqbal, M., Seadawy, A., Lu, D.: Construction of bright–dark solitons and ion-acoustic solitary wave solutions of dynamical system of nonlinear wave propagation. Modern Phys. Lett. A **34**, 1950309 (2019b)

Iqbal, M., Seadawy, A., Khalilc, O.H., Lu, D.: Propagation of long internal waves in density stratified ocean for the (2+1)-dimensional nonlinear Nizhnik–Novikov–Vesselov dynamical equation. Results Phys. **16**, 102838 (2020)

Ismael, H.F., Bulut, H.: Multi soliton solutions, M-lump waves and mixed soliton-lump solutions to the Sawada-Kotera equation in (2+1)-dimensions. Chin. J. Phys. **71**, 54–61 (2021)

Ismael, H.F., Murad, M.A.S., Bulut, H.: Various exact wave solutions for KdV equation with time-variable coefficients. J. Ocean Eng. Sci. **7**(2022), 409–418 (2022a)

Ismael, H.F., Okumus, I., Akturk, T., Bulut, H. and Osman, M.S.: Analyzing study for the 3D potential Yu–Toda–Sasa–Fukuyama equation in the two-layer liquid medium. J. Ocean Eng. Sci. (2022b)

Kirci, O., Akturk, T., Bulut, H.: Simulation of wave solutions of a mathematical model representing communication signals. J. Inst. Sci. Technol. **11**, 3086–3097 (2021)

Kudryashov, N.A.: The Painlevé approach for finding solitary wave solutions of nonlinear non-integrable differential equations. Optik **183**, 642–649 (2019)
Lu, D., Seadawy, A., Iqbal, M.: Mathematical methods via construction of traveling and solitary wave solutions of three coupled system of nonlinear partial differential equations and their applications. Results Phys **11**(2018), 1161–1171 (2018)
Munro, S., Parkes, E.: The derivation of a modified Zakharov–Kuznetsov equation and the stability of its solutions. J. Plasma Phys. **62**, 305–317 (1999)
Naher, H., Abdullah, F.A.: The improved (G′/G)-expansion method for the (2+1)-dimensional modified Zakharov–Kuznetsov equation. J. Appl. Math. (2012). https://doi.org/10.1155/2012/438928
Nisar, K.S., Ilhan, O.A., Abdulazeez, S.T., Manafian, J., Mohammed, S.A., Osman, M.S.: Novel multiple soliton solutions for some nonlinear PDEs via multiple Exp-function method. Results Phys. **21**, 103769 (2021)
Osman, M.S., Tariq, K.U., Bekir, A., Elmoasry, A., Elazab, N.S., Younis, M., Abdel-aty, M.: Investigation of soliton solutions with different wave structures to the (2 + 1)-dimensional Heisenberg ferromagnetic spin chain equation. Commun. Theor. Phys. **72**(3), 035002 (2020)
Park, C., Khater, M.M.K., Abdel-Aty, A.H., Attia, R.A.M.M., Lu, D.: On new computational and numerical solutions of the modified Zakharov–Kuznetsov equation arising in electrical engineering. Results Phys. (2020a). https://doi.org/10.1016/j.rinp.2020.103317
Park, C., Nuruddeen, R.I., Ali, K.K., Muhammad, L., Osman, M.S.: Novel hyperbolic and exponential ansatz methods to the fractional fifth-order Korteweg–de Vries equations. Adv. Differ. Equ. **1**, 627 (2020b)
Saliou, Y., Abbagari, S., Houwe, A., Osman, M.S., Yamigno, D.S., Crepin, T.K., Inc, M.: W-shape bright and several other solutions to the (3+1)-dimensional nonlinear evolution equations. Mod. Phys. Lett. B **35**(30), 2150468 (2021)
Seadawy, A.R.: Three-dimensional nonlinear modified Zakharov–Kuznetsov equation of ion-acoustic waves in a magnetized plasma. Comput. Math. Appl. **71**, 201–212 (2016a)
Seadawy, A.R.: Three-dimensional nonlinear modified Zakharov–Kuznetsov equation of ion-acoustic waves in magnetized plasma. Comput. Math. Appl. **71**, 201–212 (2016b)
Seadawy, A., Lu, D., Iqbal, M.: Application of mathematical methods on the system of dynamical equations for the ion sound and Langmuir waves. Pramana-J. Phys. **93**(1), 1850217 (2019a)
Seadawy, A., Iqbal, M., Lu, D.: Applications of propagation of long-wave with dissipation and dispersion in nonlinear media via solitary wave solutions of generalized Kadomtsev–Petviashvili modified equal width dynamical equation. Comput. Math. Appl. **78**(2019), 3620–3632 (2019b)
Seadawy, A., Iqbal, M., Lu, D.: Nonlinear wave solutions of the Kudryashov–Sinelshchikov dynamical equation in mixtures liquid-gas bubbles under the consideration of heat transfer and viscosity. J. Taibah Univer. Sci. **13**(1), 1060–1072 (2019c)
Seadawy, A., Iqbal, M., Lu, D.: Propagation of long-wave with dissipation and dispersion in nonlinear media via generalized Kadomtsive–Petviashvili modified equal width-Burgers equation. Indian J. Phys. **94**(5), 675–687 (2020a)
Seadawy, A., Iqbal, M., Lu, D.: The nonlinear diffusion reaction dynamical system with quadratic and cubic nonlinearities with analytical investigations. Int. J. Mod. Phys. B **34**(9), 2050085 (2020b)
Seadawy, A., Iqbal, M., Lu, D.: Propagation of kink and anti-kink wave solitons for the nonlinear damped modified Korteweg–de Vries equation arising in ion-acoustic wave in an unmagnetized collisional dusty plasma. Phys. a: Stat. Mech. Appl. **544**, 123560 (2020c)
Seadawy, A., Iqbal, M., Althobaiti, S., Sayed, S.: Wave propagation for the nonlinear modified Kortewege–de Vries Zakharov–Kuznetsov and extended Zakharov–Kuznetsov dynamical equations arising in nonlinear wave media. Opt. Quantum Electron. **53**(2), 1–20 (2021)
Siddique, I., Jaradat, M.M.M., Zafar, A., Mehdi, K.B., Osman, M.S.: Exact traveling wave solutions for two prolific conformable M-Fractional differential equations via three diverse approaches. Results Phys. **28**, 104557 (2021)
Sobhanian, S., Soorani, M., Khorram, S. (2003) Investigation of the solutions of the modified Zakharov–Kuznetsov equation. In: Published on the 30th International Conference on Plasma Science, ICOPS 2003. IEEE Conference Record—Abstracts
Yu, J., Wang, D.S., Yongli Sun, Y., Wu, S.: Modified method of simplest equation for obtaining exact solutions of the Zakharov–Kuznetsov equation, the modified Zakharov–Kuznetsov equation, and their generalized forms. Nonlinear Dyn. **85**, 2449–2465 (2016). https://doi.org/10.1007/s11071-016-2837-7
Zhou, Q.: Soliton and soliton-like solutions to the modified Zakharov–Kuznetsov equation in nonlinear transmission line. Nonlinear Dyn. **83**, 1429–1435 (2016). https://doi.org/10.1007/s11071-015-2415-4